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EFFECT OF WALL-TEMPERATURE DISTRIBUTION ON HEAT TRANSFER IN CENTRIFUGAL FLOW IN THE GAP BETWEEN PARALLEL ROTATING DISKS

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The effect of a radial change in the wall temperature on heat transfer in centrifugal turbulent flow in a gap between parallel rotating disks is investigated. Cases of positive, approximately constant, and negative radial gradients of the disk temperature are considered. The results of calculating by the integral method are in good agreement with the known experiments. It is shown that the change in the wall temperature has an insignificant effect in the region of the source and a determining effect in the Ekman layers, finally giving rise to a zone with return heat flux at the periphery of the cavity with negative and approximately constant gradients of the disk temperature.

Introduction. The end disk surfaces of the rotors of gas turbines are often cooled by radial blowing [1, 2]. In this case, air enters the cavity between the disks near the axis of rotation and moves radially toward the periphery.

We investigate the effect of the distribution of the temperature of a blown wall on the local Nusselt number. Experimental data [3, 4] and the results of calculating by the integral method of [5, 6] are compared. The cases of positive, approximately constant, and negative dT_w/dr on the upstream disk for an axial air supply to the cavity under the conditions $\beta_i = 0$ are considered.

Structure of Flow in the Gap. The experiments generalized in monograph [3] show that when the entry of the flux into the cavity is axial the structure of flow has the following form (Fig. 1). The entering air (totally or partially) in the form of an impact jet strikes the downstream disk and after that moves radially outward in the form of an annular wall jet. The latter generally contains more than half or the entire entering air, but this situation does not prevail to the end of the cavity. In the radial coordinate $r = r_e$ on the downstream disk, part of the air is released from the wall jet and is finally drawn in by the boundary layer on the upstream disk.

In the regions $r < r_e$ on the upper and lower disks, boundary layers develop that draw in air from the flow core. When $r > r_e$ the entire air is drawn into the so-called Ekman boundary layers [3, 4] with a constant flow rate $\dot{m}_d$ in them. With the radial entry of the flux, $\dot{m}_d = 0.5\dot{m}$ = const in each layer. With the axial entry in the region of the impact jet the fraction of the air on the downstream disk is larger than the fraction on the upstream disk. However, as has been indicated above, the air between the disks is redistributed even before the development of the Ekman layers, in which the condition $\dot{m}_d = 0.5\dot{m}$ = const on both disks still holds true [3].

In the region of the suction boundary layers, the core temperature T_∞ is constant and is equal to T_i. In the region of the Ekman layers, the suction of cold air from the core into the boundary layers ceases, and the latter, conversely, begin to release heat to the core. This leads to an increase in the temperature T_∞, which becomes itself one of the unknowns.

Integral Method. Integral equations of dynamic and thermal boundary layers have the form:

$$\frac{d}{dr}\left[r\int_0^\delta v_r\,(v_{r,\infty} - v_r)\,dz\right] + r\,\frac{dv_{r,\infty}}{dr}\int_0^\delta (v_{r,\infty} - v_r)\,dz - \int_0^\delta (v_{\varphi,\infty}^2 - v_\varphi^2)\,dz = r\tau_{wr}/\rho\,,$$

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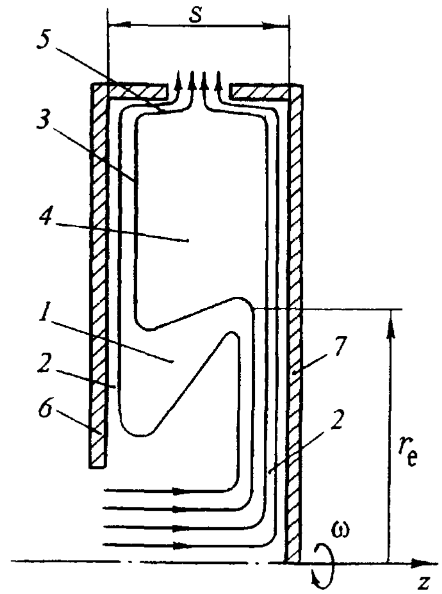


Fig. 1. Distribution of streamlines in rotating cavity with axial air supply ($\beta_i < 1$): 1) region of source; 2) suction boundary layer; 3) Ekman layer; 4) internal flowless rate core; 5) region of outlet from gap; 6) upstream disk; 7) downstream disk.

$$\frac{d}{dr}\left[r^2 \int_0^{\delta} v_r (v_\varphi - v_{\varphi,\infty})\, dz\right] + r\delta \int_0^1 v_r\, d\xi \frac{d}{dr}(r v_{\varphi,\infty}) = - r^2 \tau_{w\varphi}/\rho ,$$

$$\frac{d}{dr}\left[r \int_0^{\delta_T} v_r (T - T_\infty)\, dz\right] + \frac{dT_\infty}{dr} r\delta_T \int_0^1 v_r d\xi_T = r q_w/(c_p \rho) .$$

The profiles of the velocity v_φ and v_r, the tangential stresses $\tau_{w\varphi}$ and τ_{wr}, the profile of the temperature T, and the local Nusselt numbers Nu are determined by the following expressions [5, 6]:

$$\bar{v}_\varphi = \xi^n , \quad \bar{v}_r = \bar{v}_\varphi\, [\kappa + (\alpha - \kappa)(1 - \xi)^2] ; \tag{1}$$

$$\tau_{wr} = - \alpha \tau_{w\varphi} , \quad \tau_{w\varphi} = - \operatorname{sgn}(1 - \beta)\, \tau_w (1 + \alpha^2)^{-1/2} ; \tag{2}$$

$$c_f/2 = \tau_w/(\rho V_*)^2 = C_n^{-2/(n+1)} \mathrm{Re}_{V_*}^{-2n/(n+1)} ; \tag{3}$$

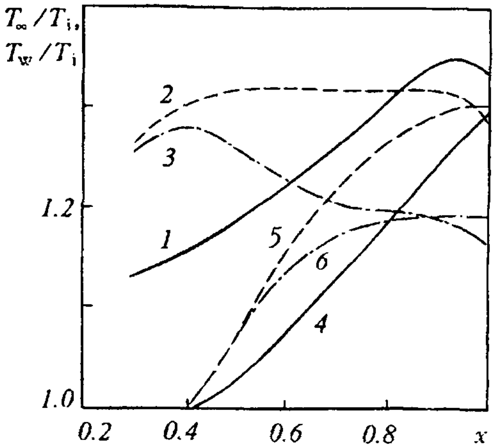
$$(T - T_w)/(T_\infty - T_w) = \xi_T^{n_T} , \tag{4}$$

$$\mathrm{Nu} = \mathrm{St} \frac{V_* r}{\nu} \mathrm{Pr} \frac{T_w - T_\infty}{T_w - T_i} ; \tag{5}$$

$$\mathrm{St} = C_n^{(n_T - 1)/(1-n)} \mathrm{Re}_{V_*}^{-n_T} (c_f/2)^{(1-n_T)/2} \Delta^{-n_T} \mathrm{Pr}^{-n_p} . \tag{6}$$

Here $\bar{v}_\varphi = (v_\varphi - \omega r)/(v_{\varphi,\infty} - \omega r)$, $\bar{v}_r = v_r/(\omega r - v_{\varphi,\infty})$, $\mathrm{Re}_{V_*} = \rho V_* \delta/\mu$, $V_* = \omega r |\beta - 1| (1 + \alpha^2)^{1/2}$.

The boundary layer equations upon integration in view of formulas (1)-(6) reduce to a form that makes it possible to numerically solve them by the Runge–Kutta method and contain the unknowns α, δ, and Δ in the



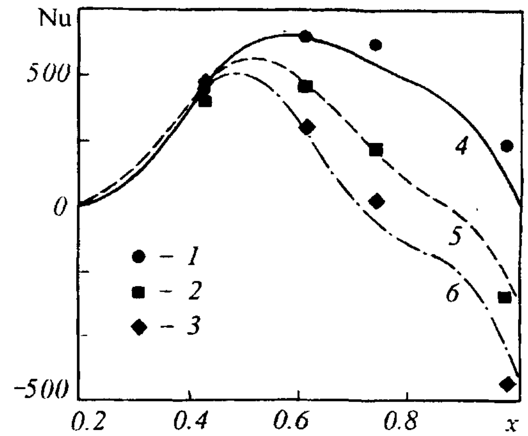


Fig. 2. Radial distribution of wall temperature T_w/T_i (curves 1-3, transformed experiments of [4]) and of flow core T_∞/T_i (curves 4-6, calculation by proposed method): 1 and 4) $dT_w/dr > 0$; 2 and 5) $dT_w/dr \approx 0$; 3 and 6) $dT_w/dr < 0$.

Fig. 3. Nusselt numbers in cavity ($Re_\varphi = 3.2 \cdot 10^6$ for cases 1 and 4, and $Re_\varphi = 3.3 \cdot 10^6$ for cases 2, 3, 5, and 6): 1-3) experiment [4]; 4-6) calculation by proposed method; 1 and 4) $dR_w/dr > 0$; 2 and 5) $dT_w/dr \approx 0$; 3 and 6)

region of the source or α, β, and T_∞ (with the constraint Δ = const) in the Ekman layers (where $v_{r,\infty} = 0$). Details of the procedure are presented in [5, 6].

In the case in question, comparison with the experiments was made at a sufficient distance from the region of the inlet to the cavity, where the authors of [3, 4] obtained experimental data for the Nusselt number. Here with a sufficient degree of accuracy we can set $v_{r,\infty} = 0$, as in the region of the Ekman layers (that is, the fraction of radial flow in the core here is small as compared to the boundary layers and has an insignificant effect on the Nu number).

Local Nusselt Number. Comparison with Experiment. Experimental temperature distributions used in the calculations are presented by Northrop and Owen [4] in the form of smoothing curves. To employ them in calculating by the integral method, we approximate these curves in the functional form of a polynomial of the 7th degree and show them in Fig. 2 in the transformed form.

The experiments [3, 4] selected for comparison were carried out for $\beta_i = 0$, $Re_\varphi = (3.2-3.3) \cdot 10^6$, $r_i/b = 0.103$, $s/b = 0.138$, $b = 0.428$ m, and $C_w = 7000$. The calculations were performed for $n = 1/10$ and $C_n = 11.5$ (C_n for the remaining values of n are given in [5, 6]); $n_p \approx 0.5$. The value $n = 1/10$ corresponds to high Re_φ numbers. The temperature profiles, as is known, are more conservative with respect to the Re_φ number, which caused the selection of the value $n_T = 1/7$.

The results of calculating the change in the core temperature T_∞ and the Nusselt numbers with positive, approximately constant, and negative gradients of T_w are shown in Figs. 2 and 3. Calculation was performed for the upstream disk (the experimental data for T_w and the Nu number are presented in [4]).

In the region of the suction boundary layers, $T_\infty = T_i$, while the Nusselt number increases as in the case of flow about a free disk. In the region of the Ekman layers, the rate of increase in the Nusselt number is retarded and subsequently begins to decrease (Fig. 3) due to an increase in the local values of T_∞ (Fig. 2). This decrease in the region of the Ekman layers is the more distinct, the smaller the difference between temperatures T_w and T_∞ due to the dissimilar distributions of T_w along the radius of the disk.

Both the experiments and the calculations by the integral method show that, for the cases $dT_w/dr \approx 0$ and $dT_w/dr < 0$, the region of negative Nusselt numbers occurs at the periphery. Physically this means that because of the high rate of decrease in the wall temperature T_w and the increase in the local air temperature T_∞ there is a region in which the temperature T_∞ becomes higher than the disk temperature T_w (Fig. 2). That is what leads to a change of sign of the heat flux: the air begins to heat up the disk rather than the reverse, as happened practically in the entire cavity.

CONCLUSIONS

1. The results of calculating by the integral method are in good agreement with the known experiments.

2. A change in the wall temperature has an insignificant effect in the region of the source and a determining effect in the Ekman layers. In this region, there ultimately occurs a zone with negative Nusselt numbers (the return direction of the heat flux) at the periphery of the cavity with negative and approximately constant gradients of the disk temperature.

NOTATION

b, radius of flux outlet from cavity, m; c_p, specific heat at constant pressure, J/(kg·K); $C_w = \dot{m}/(\mu b)$, dimensionless mass radial flow rate through cavity; $\dot{m}_d$ and $\dot{m}$, mass flow rate through boundary layer and cavity as a whole, kg/sec; $Nu = q_w r/[\lambda(T_w - T_i)]$, local Nusselt number; $Pr = \mu c_p/\lambda$, Prandtl number; q_w, heat flux on the wall, W/m^2; r, φ, and z, radial, tangential, and axial coordinates; $Re_\varphi = \rho\omega b^2/\mu$, rotational Reynolds number; s, width of gap between disks, m; $St = q_w/[\rho V_* c_p(T_w - T_\infty)]$, Stanton number; T, temperature, K; v_r, v_φ, and v_z, radial, tangential, and axial velocities, m/sec; $x = r/b$, dimensionless radial coordinate; α, tangent of swirl angle on wall; $\beta = v_{\varphi,\infty}/(\omega r)$, dimensionless tangential velocity in core of flux; δ and δ_T, thicknesses of dynamic and thermal boundary layers, m; $\Delta = \delta_T/\delta$, relative thickness of thermal boundary layer; $\kappa = v_{r,\infty}/(\omega r - v_{\varphi,\infty})$, tangent of swirl angle in flow core; λ, thermal conductivity, W/(m·K); μ, dynamic-viscosity factor, Pa·sec; $\xi = z/\delta$ and $\xi_T = z/\delta_T$, dimensionless normals to coordinate surface; ρ, density, kg/m^3; $\tau_\varphi = \mu\partial v_\varphi/dz$, $\tau_r = \mu\partial v_r/dz$, tangential and radial tangential friction stresses, Pa; ω, angular rotational velocity, 1/sec. Subscripts: w, wall; ∞, external boundary of boundary layer; i, inlet to cavity; e, boundary between source region and Ekman layers; d, disk; T, thermal; f, friction.

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